1969

A Set of 74 Test Functions for Nonlinear Equation Solvers

John R. Rice
Purdue University, jrr@cs.purdue.edu

Report Number:
69-034

A SET OF 74 TEST FUNCTIONS FOR NONLINEAR EQUATION SOLVERS

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CSD TR 34

April 1969

ABSTRACT

This report gives 74 functions appropriate to use to test programs which solve one nonlinear equation in one real variable. The functions are given in a Fortran subroutine as a two-dimensional array.

A SET OF 74 TEST FUNCTIONS FOR NONLINEAR EQUATION SOLVERS

INTRODUCTION AND SUMMARY: This report gives 74 functions in FORTRAN code which have been gathered as a set of test functions for a polyalgorithm to solve F(x) = 0. These functions are arranged in a two-dimensional array whose indices are passed through the block common

```
COMMON /FDATA/ J,JJ
```

and thus F(x) can be used in its simplest form. The ranges of the indices are

J = 1	JJ = 1 to 14
J = 2	JJ = 1 to 12
J = 3	JJ = 1 to 10
J = 4	JJ = 1 to 24
J = 5	JJ = 1 to 14

These functions may be classified into overlapping groups according to various properties the functions possess. This classification is given below with the notation that 3-8 denotes the eighth element of the third group (i.e., J = 3, JJ = 8).

SIMPLE (25 members)

1- 7	1-11	2- 2	2- 5	2- 6	3- 3	3- 6
4- 2	4- 3	4- 7	4- 9	4-10	4-11	4-12
4-14	4-15	4-17	4-18	4-19	4-20	4-21
4-22	4-23	4-24				

CLUSTERED ROOTS (7 members)

1- 6	1-10	5- 1	5- 2	5- 3	5- 4	5- 5

MULTIPLE ROOTS (14 members)

1- 2	1- 3	1- 4	1- 5	1- 6	1- 9	1-10
4- 1	4- 4	4- 5	4- 6	4- 8	4-16	5- 8

FRACTIONAL ORDER ROOTS (7 members)

1- 3	1-10	2- 8	3- 4	3- 7	5-11	5-12

DISCONTINUITIES (4 members)

1- 5	2- 9	2-10	3- 2

ASYMPTOTES TO ZERO (6 members)

1-12	5- 6	5- 7	5- 8	5- 9	5-10

ROUND OFF SENSITIVE (5 members)

1- 8	1- 9	2- 4	2- 7	3- 5

NON-FUNCTIONS (3 members)

3- 8	3- 9	3-10

PATHOLOGICAL (11 members)

1- 3	1- 5	2- 1	2- 4	2- 9	3- 1	3- 2
3- 4	4-13	5-11	5-12			

BADLY SCALED (6 members)

1- 1	1-11	2- 1	2- 3	5-13	5-14

FROM 'REAL' PROBLEMS (2 members)

1-13	2- 7

(A number of other test functions from 'real' problems are available, but they require too much Fortran code for inclusion here).

```
      FUNCTION F(X)
      COMMON/ FDATA/ J,JJ
      DATA PI /3.1415926 /
      GO TO(1 0,200,300,400,500), JJ
C                    TEST FUNCTIONS IN USE SUMMER 1967
  100 CONTINUE
      GO TO(1,2,3,4,5,6,7,8,9,10,11,12,13),J
1     F = (X-.12[ E5)*(X+.21E+8)*.1E-9
      RETURN
2     IF(ABS(X).GT.1.E9) X = 1.E8
      F = ABS(X-9.)**4.5
      RETURN
3     F = ABS(X-361.2)**.7*ABS(X+157.3)**1.5*((X-.1E-05)/ABS(X-.1E-10))
      RETURN
4     XALOG = 0.
      IF(X.NE.0.) XALOG = X*ALOG(ABS(X))
      F = (X-11.257)**3*(50.*X-.1*XALOG**2 -7.)
      RETURN
5     CONTINUE
      IF(ABS(X).GE.0.9) GO TO 501
      IF(X.LE.-.4) F = SIN(6.*X)
      IF(X.GE.-.4) F = .1
      IF(X.GE.-.1E-08) F = (X+.2E-08)/X*(X-.2E-08)/X
      IF(X.GE.-1.E-09) F = (X+.00001)*(1.+COS(400.*X)/2.)
      IF(X.GE.0.) F = ABS(X-.2)**.4*X
      IF(X.GE..5) F = ABS(X-.4)**2
501   IF(ABS(X).GE.0.9) F = ABS(X)
      IF(X.GT.313..AND.X.LT.403.) F = (X-300.)*(X-314.)*(X-399.)**2
      RETURN
6     F = ABS(X-17.)**2*ABS(X-17.1)**1.8*(X-20.)
      RETURN
7     F = (X-1.)*(X-2.)*(X-3.)*(X-4.)*(X-5.)*(X-6.)
      RETURN
8     F = ((((X-21.)*X+175.)*X-735.)*X+1624.)*X-1764.)*X+720.
      RETURN
9     Y = X + 1.11111
      F =-1. - Y*(1[ .-Y*(54.-Y*(12.-Y)))
      RETURN
1[    Y = X - 1312.
      IF(ABS(Y).GT..15+7) Y = SIGN(.1E+7,Y)
      F = ALOG10(1.+Y**2)*(Y+16.)*SQRT(ABS(Y-8.))
      RETURN
11    Y = SQRT(ABS(X-300.))
      IF(ABS(X).LE.400.) GO TO 1110
      F = ALOG10(97.)*COS(10.0 )+(X-400.)*(1.+3.*(X-400.))
      RETURN
1110  F = ALOG10(1.+Y**2)*COS(X/40.)
      RETURN
12    F = 1./(ABS(X*X*X)+1.)
      RETURN
C                    FORESTRY - INVESTMENT RETURN FUNCTION
13    Y = 1. + X
```

```
      F = 20./Y**15 +36./Y**25 +40./Y**33 +475./Y**40 -1.12*(Y**40-1.)
     ) /(X*Y**40) -4.5 -6./Y**4 -2./Y**8
      RETURN
C                    TEST FUNCTIONS OF SEPT. 1967
  290 GOTO(20,21,22,23,24,25,26,27,28,29,290,291),J
   20 IF(ABS(X) .LT. 1.E+8) GO TO 101
      F=  1.222E+9 - ABS(X)
      RETURN
  101 F = 1.+ X**4
      RETURN
   21 F = SIN(X)
      RETURN
   22 F = COS(.001*X)
      RETURN
   23 F = (X+1.)*(X+5.)*(X-27.)*(X-109.)
      IF( ABS(F).LT. 1.E-5*ABS(X) ) F =13.
      RETURN
   24 DO 607 I = 1,40
  607 DU =  EXP(5.)*(DU+1.)/(ABS(DU)+COS(2))
      Y = ABS(X)
      F= SIN(X)*(X**2+1.) - EXP(SQRT(Y)/10.0*(X-1.)*(X**2-5.)
      RETURN
   25 IF( ABS(X) .GT. 6500.) GO TO 601
      F = SIN(X)*(X**2 +1.) - EXP(SQRT(ABS(X)))*(X-1.)*(X**2-5.)
      RETURN
  601 F = 1.E+30*SIGN(1.,X-5.0)
      RETURN
C                    MINIMIZE INTEGRAL  EXP(XT)(T*T+1) - (T**3+2) ON (0,1)
   26 I700 = 1
      Z = X*(1.000000) - 1.E-5*SIGN(1.,X)
  702 G=0
      DO 701 I = 2,40
      T = FLOAT(I-1)/40.
  701 G = G + ( EXP(Z*T)*(T**2+1.) -(2.+T**3))**4
      G =(2.*G +  1. + (EXP(Z)*2.-3.)**4)/80.
      GO TO (703,705), I700
  703 G1 = G
      Z = X*(1.00001) +1.E-5*SIGN(1.,X)
      I700 = 2
      GO TO 702
  705 F  =(G - G1)/(2.E-5*(1.+ABS(X)))*SIGN(1.,X)
      RETURN
   27 F = (ABS(X-999.))**.4
      RETURN
   28 F =(X-3.)/(X-4.)*(X-5.)/(ABS(X-6.)**.6*SIGN(1.,X-6.))*(X*X+1.)
      RETURN
   29 IF( ABS(X) .LT. 10. ) GO TO 1010
      F = ABS(X+5.)**2
```

```
      RETURN
 1010 F = SIGN(1.+X*X,SIN(X))
      RETURN
  200 F = SIGN(1.+X*X,SIN(X))
      RETURN
  201 IF(  ABS(X) .GT. 88.0 )   X = 88.0*SIGN(1.0,X)
      F = EXP(-X)*(1.+.5*SIN(X))
      RETURN
C
C          FUNCTIONS OF FEB. 1968
  300 GOTO(30,31,32,33,34,35,36,37,38,39),J
   30 F=1.
      IF( X.GT.1000. .AND. X.LT. 1002. )   F= (X-1001.)*2.-1.
      RETURN
   31 IF( X.LT. -3.)  F=0.
      IF( X.GE. -3. .AND. X.LT.-2.) F=X+3.
      IF( X.EQ. -2.)  F= -1.
      IF( X.GT. -2. .AND. X.LT.-1.) F=1.
      IF( X.GE. -1. .AND. X.LT.0. ) F=2.*X*X -1.



      IF( X.GE. 0.  .AND. X.LT.2. ) F=X-1.
      IF( X.GE. 2.  .AND. X.LT.4. ) F=X-3.
      IF( X.GE. 4. )  F=COS(X-4.)
      RETURN
   32 F = SIN(PI/(AMAX1(1.E-12,ABS(X))*SIGN(1.,X)))*X
      RETURN
C                                                  DIST(X,1,1/10, 1/100,--- )
   33 Y=-X
      IF(Y.GE.0.)  GOTO 916
      Y=X-1.
      IF(Y.GE.0.)  GOTO 916
      A=.1
  910 Z=Y
      Y= X-A
      IF(Y.GE.0.)  GOTO 914
      A = .1*A
      GO TO 910
  914 IF( Y.LE. -Z)GOTO 916
      Y=-Z
  916 F=Y
      RETURN
   34 Z = AMAX1(1.E-20,ABS(X))
      F=X-1./SQRT(1.+1./(Z*Z))
      RETURN
   35 F= (X+1.)/(X*X+2. )
      RETURN
C                                                  DIST(X,1,10,100,--- )
   36 Y=1.-X
      IF( Y.GE.0. ) GO TO 714
```

```
      A=10.
  715 Z=Y
      Y=A-X
      IF( Y.GE.0. ) GO TO 712
      A=A+10.
      GOTO 715
  712 IF(Y .LE.-Z ) GO TO 714
      Y= -Z
  714 F=Y
      RETURN
   37 F = SIN(PI/SQRT(ABS(X)))*Y
      RETURN
   38 F=0.
      RETURN
   39 X=X+1.
      F=0.
      RETURN
C
C     PROBLEMS TAKEN FROM TRAUB(0-5),RALSTON(6-15),WILDERANDT(16-23)
  400 CONTINUE
  401 GOTO( 40 ,41,42,43,44,45,46,47,48,49,50,51,52,53,54,55,56,57,58,59
     *        ,60,61,62,63,64),J
   40 F =  (X*(X-1.))**2
      RETURN
   41 F =   X*X - 1.
      RETURN
   42 F =   X**20 - 1.
      RETURN
   43 F =   EXP(X)*(X-1.)**2
      RETURN
   44 F =   X*(X-1.)**2
      RETURN
   45 F =   EXP(X)*(X-1.)**2
      RETURN
   46 F =   SIN(X) - X/2.
      RETURN
   47 F =   (SIN(X)- X/2.)**2
      RETURN
   48 F =   X*EXP(-X)
      RETURN
   49 F =   (((4.*X -5.)*X + 9.)*X - 18.)
      RETURN
   50 F =   ((X-5.)*X + 17.)*X + 21.
      RETURN
```

```
   51 F =    (((X-3.)*X -24.)*X -150.)*X -100.
      RETURN
   52 F=1.
      DO 451 LL = 1,20
  451 F = F*(X-FLOAT(LL))
      RETURN
   53 F =    COS(X) - X*EXP(X)
      RETURN
   54 F =    TAN(X) - COS(X) -.5
      RETURN
   55 F =    (COS(X)- X*EXP(X))**3
      RETURN
   56 IF( X .EQ. 0. )   X = 1.E-250
      F =    X + ALOG(ABS(X))
      RETURN
   57 F =    (X*X -1.)*X -1.
      RETURN
   58 F =    X - EXP(-X)
      RETURN
   59 F =    TAN(X) - 1.01*X
      RETURN
   60 F =    TAN(X) - 2.*X
      RETURN
   61 F =    TAN(X) - 30.*X
      RETURN
   62 F =    X**3 -2.*X -5
      RETURN
   63 F =    X*(X-3.) -4.*(SIN(X))**2
   64 CONTINUE
      RETURN
C
C          FUNCTIONS OF FEBRUARY 1967
  500 CONTINUE
      GO TO(70,71,72,73,74,75,76,77,78,79,80,81,82,83) J
   70 Y = X-113.
  570 F = SIN(Y)*(1. +Y*Y) - EXP(SQRT(ABS(Y))/10.)*(Y-1.)*(Y*Y-5.)
      RETURN
   71 Y = X + 9423.
      IF( ABS(Y) .GT. 2.5E+6 ) Y = SIGN(2.5E+6,Y)
         GO TO 570
   72 Y = X+3.
      IF( ABS(Y) .GT. 1. )     GO TO 572
      F = 2. *COS(6.*PI * Y)
      RETURN
  572 F = 1 + Y*Y
      RETURN
   73 Y = 10.*(X+6.2)
      IF( ABS(Y) .GT. 1. )     GO TO 573
      F =  2.*COS(9.*PI *Y)
      RETURN
  573 F = (1.+Y*Y)*(1.+(Y-1.)*(3.+(Y-1.)*(-2.-Y)))
      RETURN
```

```
 74 F = (X-1.1)*(X-1.11)*(X-2.)*(X-2.002)*(X-2.00002)*(X-4.)*(X-4.05)
    *    *(X-2.9()
      RETURN
 75 Y= ABS(X)
      IF( Y .GT. 500. )        GO TO 575
      F = EXP(-Y)
      RETURN
575 F = .1E-175
      RETURN
 76 Y= ABS(X)
      IF( Y .GT. 10000.)       GO TO 576
      F = (X-1.)*(X+2.4)*EXP(-Y)
      RETURN
576 F = .1E-175
      RETURN
 77 F = (X-2.)**2*(X+10.)*(X-252.)/(X**6*(1.+.9*SIN(X))+.001)*1.E+3
      RETURN
 78 F = (X-400.)*(X+2(02.)*(X+2)*(X-10000.)*.1E+5/
    *      ((X**2 +1.)*(1.+.2*COS().)))
      RETURN
 79 F = (X-2.)*(X+6.)*1.E+4/(1.+X**12.)
      RETURN
 80 IF( X.GT. 2.)      F = YX-2.)*(1.+X*X/EXP(SQRT(X)))
      IF( X.LT.2..AND.X.GT.1.)    F = SQRT(ABS(X-2.)*ABS(X-1.))
      IF(X.LT.1..AND.X.GT.0.)     F = X**2.2*(1.-X)**1.4
      IF(X.LT.0..AND.X.GT.-1.)    F = (-X)**1.4*(X+1.)**.6
      IF(0.LE.-1.  )              F = (-1.-X)**1.8*SIN(1./SQRT(-1.-X))
      RETURN
 81 IF(X.GT.-1.)    F = ABS(X)**1.4*(X+1.)**.6
      IF(X.LE.-1.)    F = (1.+X)**2*(1. + .98*SIN(1./SQRT(-1.-X)))
      RETURN
 82 Y = 400.*(X-.01)
      IF( ABS(Y) .LT. 1. )     F =(Y+.2)*(Y-.3)
      IF( ABS(Y) .GE. 1. )     F = .14*(1.+SIN(Y)+.1*Y*Y)
      RETURN
 83 Y = 1.E+9*(X-1.E-9)
      F = (Y-4.)*(Y+2.)*(Y+41.)
      RETURN
      END
```